

**Exercise 22**

Find the derivative of the function.

$$y = \left(x + \frac{1}{x}\right)^5$$

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**Solution**

Take the derivative using the chain rule.

$$\begin{aligned}y' &= \frac{dy}{dx} = \frac{d}{dx} \left[ (x + x^{-1})^5 \right] \\&= 5 (x + x^{-1})^4 \cdot \frac{d}{dx} (x + x^{-1}) \\&= 5 (x + x^{-1})^4 \cdot (1 - x^{-2}) \\&= 5 \left(x + \frac{1}{x}\right)^4 \left(1 - \frac{1}{x^2}\right) \\&= 5 \left(\frac{x^2 + 1}{x}\right)^4 \left(\frac{x^2 - 1}{x^2}\right) \\&= \frac{5}{x^6} (x^2 + 1)^4 (x^2 - 1)\end{aligned}$$